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# Leveraged Borrowing and Boom-Bust Cycles\*

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## Abstract

Investment booms and asset "bubbles" are often the consequence of heavily leveraged borrowing and speculations of persistent growth in asset demand. We show theoretically that dynamic interactions between elastic credit supply (due to leveraged borrowing) and persistent credit demand (due to consumption habit) can generate a multiplier-accelerator mechanism that transforms a one-time productivity or financial shock into large and long-lasting boom-bust cycles. The predictions are consistent with the basic features of investment booms and the consequent asset-market crashes led by excessive credit expansion.

*Keywords:* Asset Bubble, Investment Boom, Borrowing Constraints, Multiplier-Accelerator, Elastic Credit Supply, Habit Formation.

*JEL codes:* E21, E22, E32, E44, E63.

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# 1 Introduction

A credit boom in the form of excessive loans unleashed from the banking sector to households and nonfinancial firms has large aggregate effects. In an empirical study of industrial and emerging countries over the period 1960-2006, Mendoza and Terrones (2008) document evidence that unusually large credit expansions go hand in hand with over-investment and a volatile economy.

The typical credit boom is associated with a first phase during which output, consumption, and especially investment rise significantly above trend, followed by large downswings below trend. According to Mendoza and Terrones (2008, p. 17), “credit booms are associated with a well-defined cyclical pattern in output and expenditures.” Two features of their empirical facts are worth noticing: (i) The economy cycles around a long-run balanced growth path and the average boom-bust cycle is at least 7 years long; (ii) during a boom-bust cycle, the volatility of consumption and output are similar but that of investment is excessive (more than 5 times larger than output). These features are in sharp contrast to regular business cycles analyzed by the traditional real business cycle (RBC) literature, where fluctuations are more random with a shorter average duration, and consumption is significantly smoother than output whereas investment is only 2-3 times more volatile than output. Similar long-swing investment booms, such as the rise and burst of the dot-com bubble, are also stressed by Schneider and Tornell (2004) and Caballero, Farhi, and Hammour (2006) as “speculative growth episodes” accompanied by extreme stock market valuations and large credit expansions with low interest rates.

In this paper, we provide a general-equilibrium model to explain the business-cycle features associated with a credit boom. In particular, we show: (i) how it is possible for an economy (whether closed or open) to transform a one-time, serially uncorrelated shock to total factor productivity (TFP) or financial conditions (such as the loan-to-collateral ratio, the interest rate, and borrower’s credit worthiness) into a large and prolonged bubble-like investment boom with standard production technologies, preferences, and a unique rational expectations equilibrium; (ii) why does the upswing eventually go bust.

Using the most recent subprime housing crisis in the United States as an example, we notice several important features of credit-driven investment booms and busts in the housing market: (i) heavily leveraged borrowing, (ii) expectations of persistent growth in housing and consumption demand, (iii) relatively low real interest rates, and (iv) the absence of significant

and persistent TFP growth (or technology innovations). These features are suggestive for our modeling strategies. Leveraged borrowing implies a financial-accelerator effect (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Highly persistent housing and consumption demand in the absence of significant income (or productivity) growth may indicate "Catching Up with the Joneses" (CUWJ) behaviors among households competing for living standards. Low real interest rates indicate substantial supply of loanable funds. Thus, our basic building blocks or assumptions include (i) elastic credit supply based on collateralized borrowing and (ii) consumption reference point—that individuals derive utility not only from the level of their current consumption, but also from how their consumption compares to their own past consumption (internal habit) or the consumption of the people around them (CUWJ or external habit).<sup>1</sup>

We embed these two assumptions into an infinite-horizon general-equilibrium model with heterogeneous agents. We find that when borrowers have strong incentives to mimic each other's living standard and lenders are willing to supply credit based on borrowers' credit worthiness, endogenous boom-bust cycles emerge and such cycles resemble the pattern documented by Mendoza and Terrones (2008). In particular, when debt endogenously builds up during the booming phase, the real interest rate remains low, and deviations of output and consumption from trend are hump-shaped but quantitatively similar to each other whereas investment swings are several times larger. In addition, the boom looks like a leveraged "bubble" because both credit and the prices of collateralized assets (e.g., housing and equity) go up significantly in the initial hump-shaped phase of the cycle and then collapse sharply below their long-run trend with a well defined endogenous turning point.

A significant change in financial conditions—such as the loan-to-collateral ratio, the interest rate—or in borrower's productivity (or credit worthiness) can trigger an initial credit boom. However, both collateralized borrowing and consumption habit are needed to support the credit-driven investment boom in our model. First, consumption habit on the borrower side generates strong incentives for saving in the initial period of a boom so as to outperform the reference point (past consumption or other people's living standard) in the longer run through wealth accumulation. This competitive saving behavior facilitates asset (capital and land) investment by smoothing households' consumption demand. It also generates persistence in aggregate consumption, which ensures firms' prospect of future sales. Second, when borrowing is constrained by the value of collateralized assets, the incentives for asset accumulation are

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<sup>1</sup>Internal and external habit formation give similar results in our model because the former acts as competition for living standards with one's own historical self. In the sequel, we use consumption habit to refer to external and internal habits.

compounded because undertaking investment improves the borrowers' credit worthiness, which relaxes their future borrowing constraints. These two motives reinforce each other dynamically, generating a cumulative process of investment and output expansion once the economy is shocked by improved financial conditions that reduce the cost of borrowing, or by good news in the borrowers' aggregate productivity that signals the borrowers' credit worthiness. In contrast, absent consumption habit, the extra income or loan obtained in the impact period would be largely consumed by households right away rather than saved, which would abort the multiplier-accelerator propagation mechanism in the model by reducing borrowers' net worth and ability to borrow in the future, leading only to a monotonic impulse response of output to shocks, despite collateralized borrowing.

Hence, CUWJ consumption is crucial in our model for generating and supporting a persistent credit boom. However, a perpetual boom with excessive investment and asset accumulation is not sustainable because a rising debt level and diminishing marginal product of assets will ultimately erode the borrowers' net worth and aggregate demand (consumption and investment expenditures), resulting in falling asset price and falling collateral value. In the downturn phase, the multiplier-accelerator propagation mechanism is reversed. Low marginal products of capital reduce investment incentives, and slowly falling consumption (due to consumption habit) leads to insufficient savings. As a result, production capacity and output level decline at an increasing speed, forcing the economy to over-shoot its steady state from above in a downturn. A contraction thus generates a recession. Nonetheless, the recession will eventually end—as the production capacity falls, the marginal product of capital will ultimately become high enough to make investment profitable again, which sets off a new round of recovery.

To summarize, the boom-bust cycles are created by an endogenous multiplier-accelerator mechanism, which translates a one-time positive shock to aggregate credit supply on the lender side, or credit worthiness on the borrower side, into large and highly persistent movements in aggregate spending and output. At the peak of the expansion, the increases in the capital stock and output are several times larger than their initial responses to the shock, and in the contraction phase, they over-shoot their long-run steady-state level from above. In this process an initial boom plants the seed for a future recession and vice versa.

*Related Literature.* Our paper belongs to the literature that explains why collateral constraints amplify shocks. More specifically, our formulation of procyclical credit supply borrows from Kiyotaki and Moore (KM 1997), who have shown that endogenous credit limits based on the value of collateralized assets lead to credit cycles. However, subsequent investigations have found that such a propagation mechanism disappears when embedded into a standard RBC model (see, for example, Kocherlakota, 2000, and Cordoba and Ripoll, 2004a; see also

Iacoviello, 2005, for a monetary model). That is, collateralized lending is not by itself sufficient for generating credit cycles in a neoclassical framework. The key is that without additional savings to provide loanable funds to lower the real interest rate and without persistent increase in consumption demand, firms do not have a strong enough incentive to invest and expand production capacity, even though doing so can relax their borrowing constraints and improve credit worthiness. Hence, additional incentives for savings and anticipated persistent consumption growth are key.

We are not the first to generate endogenous boom-bust cycles in such a framework. KM (1997) and Cordoba and Ripoll (CR, 2004b) also show that it is possible to generate endogenous boom-bust cycles by adding certain forms of investment adjustment costs into the basic KM model. The intuition is that when entrepreneurs (borrowers) are not able to invest to the full or desired amount within a single period due to adjustment costs, they opt to postpone or spread out investment across multiple periods. This generates lagged demand for investment. In this regard, our model is similar to theirs. However, both the KM (1997) model and the CR (2004b) model rely on linear technologies and preferences and also on a constant savings rate to generate boom-bust cycles.<sup>2</sup> In addition, the magnitude and length of cycles in their model are not quantitatively consistent with the data. For example, the magnitude of the cycle in KM (1997) is too small and the cycles in CR (2004b) are too long (about 15 years or longer) compared to the data reported in Mendoza and Terrones (2008).

The crucial distinction between our approach and the existing literature is that we use standard technologies and preferences in the RBC literature and we focus on the role of consumption inertia in creating boom-bust business cycles. A key stylized fact of the business cycle is that lagged consumption forecasts aggregate output while lagged capital investment does not. For example, there exists a one-directional "causal" relationship among aggregate consumption, output, and business investment: "[p]ostwar U.S. data show that consumption growth 'Granger-causes' gross domestic product (GDP) growth but not vice versa and that GDP growth in turn Granger-causes business investment growth but not vice versa" (Wen, 2007, p195). This fact suggests that lagged consumption contains information about future output and investment which is not available from the past history of output and investment. An intuitive explanation is that firms undertake fixed investment only after observing increases in consumption demand that are expected to persist in the future. Otherwise, without observing persistent high consumption demand forthcoming in the future, firms could simply increase

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<sup>2</sup>A recent paper by Liu, Wang, and Zha (2011) studies business-cycle comovements between land prices and business investment in a model similar to ours. Their model does not generate the type of boom-bust cycles emphasized in this paper.

output temporarily by a higher capacity utilization rate instead of expanding production capacity by undertaking costly investment. This understanding leads us to focus on persistent consumption demand (through consumption habit or CUWJ) as a key driver of the business cycle.

Although its importance in understanding asset returns and consumption behaviors has been well acknowledged in the literature, the role of consumption habit in generating boom-bust cycles has not been fully appreciated nor thoroughly analyzed.<sup>3</sup> In the macroeconomics literature, habit persistence has been employed to explain asset pricing puzzles (Boldrin, Christiano and Fisher, 2001) and the positive relationship between savings and growth (Carroll, Overland and Weil, 2000).<sup>4</sup> But these models do not emphasize hump-shaped boom-bust cycles. In this paper we pursue this line of research further by showing that dynamic interactions between consumption habit (on the demand side of credit) and collateralized lending (on the supply side of credit) create powerful credit cycles featuring excessive credit lending and over-investment. This result is obtained despite strongly diminishing returns to investment and agents being risk averse, in contrast to KM (1997) and CR (2004b).

Does consumption habit (or CUWJ) reflect actual preferences? John Stuart Mill once observed that “men do not desire to be rich, but richer than other men.”<sup>5</sup> This common notion has been confirmed by many empirical studies. For example, Luttmer (2005) investigates whether individuals feel worse off when others around them earn more. Using a sample of social surveys for self-reported happiness, he finds that, controlling for an individual’s own income, higher earnings of neighbors are associated with lower levels of self-reported happiness. Using a unique data set on suicide death, Daly, Wilson and Johnson (2008) find strong empirical evidence supporting the notion that individuals care about the incomes of both those above them and those below them in their utilities.

Perhaps the closest related empirical evidence of consumption habit is provided by the work of Ravina (2007). Ravina tests the micro story behind consumption habit models by looking into actual household consumption decisions and estimating a consumption Euler equation for a sample of U.S. credit-card holders. The estimation incorporates both internal and external habit motives. In particular, Ravina measures the external habit of each household by the consumption level of the city in which the household lives. Ravina finds very strong and unambiguous evidence of consumption habit at the household level—the combined internal and

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<sup>3</sup>In the asset pricing literature, habit preference has been used to explain the equity premium puzzle (see, e.g., Constantinides, 1990; Abel, 1990; Campbell and Cochrane, 1999).

<sup>4</sup>For the early literature on the relationship between consumption habits and cycles, see Ryder and Heal (1973) and their followers.

<sup>5</sup>This quote is taken from Luttmer (2005).

external habit coefficient is above 0.8.

Finally, the literature on business cycles with credit market imperfections has shown how financial frictions may generate hump-shaped output dynamics.<sup>6</sup> Our paper complements these existing studies, as we show that credit market frictions, when interacted with competition for living standards, create not only hump-shaped dynamics but also highly persistent dampened cycles. Proving the presence of cycles is important because it frees the RBC approach from relying on technological regress (that is, negative TFP shocks) to generate recessions after credit booms. In this sense, our paper fits within the recent literature with financial frictions and habits (see for example Christiano et al, 2010, Gertler, Kiyotaki and Queralto, 2010, Iacoviello and Neri, 2010).

Closely related is a strand of the literature showing that boom-bust patterns occur when credit constraints create multiple equilibria (for example Schneider and Tornell, 2004, Caballero, Farhi, and Hammour, 2006)<sup>7</sup>. In such an approach, although the boom may be permanent, it is a fragile equilibrium in the sense that the economy might jump to a lower equilibrium and stay there. However, the mechanism that may end the boom is left outside the model, in contrast with our setting with a unique cyclical equilibrium around the steady state.

In what follows, Section 2 presents a benchmark model of credit cycles with reproducible capital. It is shown that this model can generate boom-bust cycles under productivity shocks. Section 3 shows the robustness of our results by considering financial shocks as well as various extensions of the benchmark model, such as small open economy with interest rate shocks, symmetric two sectors and elastic labor supply. Section 4 studies some implications for stabilizing policies, and Section 5 concludes the paper with remarks for future research.

## 2 The Benchmark Model

### 2.1 Structure

There are two types of agents in the economy, lenders and borrowers. Lenders do not produce, but provide loans (credit) to borrowers. In this sense, lenders serve the role of banks or financial intermediaries in the economy.<sup>8</sup> The type of credit provided by lenders are one-period loans that can be used to finance consumption and investment. Lenders derive utilities from consumption

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<sup>6</sup>As exemplified by the contributions of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Krishnamurthy (2003), Cooley, Marimon, and Quadrini (2004), Iacoviello (2005), Campbell and Hercowitz (2006), Boháček and Rodríguez Mendizábal (2007), and Christiano, Motto, and Rostagno (2007) among many others.

<sup>7</sup>Also see Wang and Wen (2009) for a recent analysis of speculative bubbles and financial crisis using a multiple-equilibrium approach.

<sup>8</sup>Section 3 shows that our main results still hold when lenders are also producers.



and land,<sup>9</sup> do not accumulate fixed capital, and use interest income (profits) from payment on previous loans to finance current consumption and land investment. The budget constraint of a representative lender is given by

$$\tilde{C}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + B_{t+1} \leq (1 + R_t)B_t, \quad (1)$$

where  $\tilde{C}_t$  denotes consumption,  $\tilde{L}_t$  the amount of land owned by the lender in the beginning of period  $t$ ,  $Q_t$  the relative price of land,  $B_{t+1}$  the amount of new loans (credit lending) generated in period  $t$ , and  $R_t$  the real interest rate. The utility function of the lender is given by

$$U_L = \frac{\tilde{C}_t^{1-\sigma_L}}{1-\sigma_L} + b \frac{\tilde{L}_t^{1-\sigma_W}}{1-\sigma_W}, \quad \{\sigma_L, \sigma_W, b\} \geq 0; \quad (2)$$

and the time discounting factor is  $\tilde{\beta} \in (0, 1)$ . Notice that lenders do not have consumption habit. Our results remain valid if the lenders are perfectly symmetric to borrowers in terms of preferences and technologies (see Section 3).

According to Mendoza and Terrones (2008, Table 6), changes in both TFP and financial conditions are the main factors that trigger credit booms. More precisely, about 50% of the credit booms in emerging economies are preceded by large capital inflows, while about 40% of credit booms in industrial countries are preceded by large TFP gains. Therefore, we use both TFP and financial shocks in our model. In the case of a closed economy, we use changes in the loan-to-collateral ratio as a proxy for financial shocks. But we will also study interest rate shocks in an open economy extension of the benchmark model in Section 3.

Borrowers can produce goods using land and capital.<sup>10</sup> The production technology is given by

$$Y_t = A_t K_t^\alpha L_t^\gamma, \quad \alpha, \gamma \in (0, 1), \alpha + \gamma < 1; \quad (3)$$

where  $A_t$  is TFP,  $L_t$  denotes the amount of land owned by the borrower, and  $K_t$  denotes capital stock. Capital is reproducible but the total amount of land is in fixed supply,

$$L_t + \tilde{L}_t = \bar{L}. \quad (4)$$

Although it is not essential, we allow land in the model for two purposes: (i) to study asset price movements and their role in affecting the collateral value; and (ii) to keep the model comparable to KM and the related literature.<sup>11</sup>

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<sup>9</sup>As in Iacoviello (2005), introducing land in the utility function is a short cut for generating a demand for assets by the lenders.

<sup>10</sup>Labor is fixed in the basic model. Elastic labor will be introduced into the model in Section 3.

<sup>11</sup>We have also experimented with a model without land and with capital serving as collateral. The results are qualitatively similar.

A representative borrower in each period needs to finance consumption ( $C_t$ ), land investment ( $L_{t+1} - L_t$ ), capital investment ( $K_{t+1} - (1 - \delta)K_t$ ), and loan payment that includes both the principal ( $B_t$ ) and the interest ( $R_t B_t$ ), where  $\delta \in (0, 1)$  is the depreciation rate of capital. The budget constraint of the borrower is given by

$$C_t + K_{t+1} - (1 - \delta)K_t + Q_t(L_{t+1} - L_t) + (1 + R_t)B_t \leq B_{t+1} + A_t K_t^\alpha L_t^\gamma. \quad (5)$$

The momentary utility function of the representative borrower is given by

$$U_B = \frac{[C_t - \rho \bar{C}_{t-1}]^{1-\sigma_B}}{1-\sigma_B}, \quad \sigma_B \geq 0; \quad (6)$$

where  $\bar{C}_t$  denotes the average consumption of the borrowers and  $\rho \in (0, 1)$  measures the strength of consumption externality.<sup>12</sup> Borrowers are assumed to be less patient than lenders; hence, their time discounting factor satisfies  $\beta < \tilde{\beta}$ .

The borrowing constraint faced by the borrower is

$$(1 + R_{t+1})B_{t+1} \leq \theta_t Q_{t+1} L_{t+1}, \quad (7)$$

where  $\theta_t$  is the loan-to-collateral ratio and reflects shocks to terms of loans or current financial conditions. For example, a positive shock to  $\theta_t$  implies that creditors are willing to lend more with the same collateral value of land. Following KM, reproducible capital does not have collateral value in our model but relaxing this assumption does not affect our results.<sup>13</sup> The borrowing constraint imposes that the amount of debt in the beginning of the next period cannot exceed a fraction  $\theta$  of the collateral value of assets owned by the borrower next period. The rationale for this constraint is that, due to lack of contractual enforceability, the lender has incentives to lend only if the loan is secured by the value of the collateral.

## 2.2 Allocation without Borrowing Constraints

In this subsection, we derive an allocation that obtains in a "first-best" environment with perfect risk sharing, absent the credit constraint (7).<sup>14</sup> We show that there is no credit cycle in such an environment with realistic parameter values even if the lender also has consumption

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<sup>12</sup>The results are similar when habit formation is internalized. We choose to present the external habit model because it is simpler. Our main result also holds under multiplicative habits, as in Abel (1990).

<sup>13</sup>If capital is firm specific, then it has little collateral value on the market.

<sup>14</sup>By "first-best" allocation we mean allocation with perfect risk sharing without borrowing constraints. The results are derived under external habit formation but are similar under internal habit formation.

habit. Denoting  $\tilde{\bar{C}}_t$  as the average consumption of the lenders in period  $t$  the allocation without borrowing constraints is equivalent to the solution to the following program

$$\max_{\{C_t, \tilde{C}_t, \tilde{L}_t\}} E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{[C_t - \rho \tilde{C}_{t-1}]^{1-\sigma_B}}{1-\sigma_B} + \tilde{\beta}^t \left[ \frac{[\tilde{C}_t - \rho \tilde{C}_{t-1}]^{1-\sigma_L}}{1-\sigma_L} + b \frac{\tilde{L}_t^{1-\sigma_W}}{1-\sigma_W} \right] \right\}$$

subject to

$$C_t + \tilde{C}_t + K_{t+1} - (1-\delta)K_t \leq A_t K_t^\alpha L_t^\gamma \quad (8)$$

$$L_t + \tilde{L}_t \leq \bar{L}, \quad (9)$$

The first-order conditions are given by

$$\beta^t [C_t - \rho C_{t-1}]^{-\sigma_B} = \tilde{\beta}^t [\tilde{C}_t - \rho \tilde{C}_{t-1}]^{-\sigma_L} \quad (10)$$

$$\tilde{\beta}^t [\tilde{C}_t - \rho \tilde{C}_{t-1}]^{-\sigma_L} = \tilde{\beta}^{t+1} [\tilde{C}_{t+1} - \tilde{C}_t]^{-\sigma_L} \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \quad (11)$$

$$\tilde{\beta}^t [\tilde{C}_t - \rho \tilde{C}_{t-1}]^{-\sigma_L} \gamma \frac{Y_t}{L_t} = \tilde{\beta}^t b \tilde{L}_t^{-\sigma_W} \quad (12)$$

In the limit, because  $\tilde{\beta} > \beta$ , equation (10) implies  $\lim_{t \rightarrow \infty} [C_t - \rho C_{t-1}]^{-\sigma_B} = 0$  provided that  $\lim_{t \rightarrow \infty} [\tilde{C}_t - \rho \tilde{C}_{t-1}] > 0$ ; which in turn implies that the borrower's consumption level goes to zero in the limit,  $\lim_{t \rightarrow \infty} C_t = 0$ .<sup>15</sup> Equation (11) gives the modified golden-rule capital-to-output ratio in the steady state,  $\frac{K}{Y} = \frac{\alpha \tilde{\beta}}{1 - \tilde{\beta}(1-\delta)}$ , where  $\tilde{\beta}$  is the inverse of the gross interest rate. The resource constraint (8) implies the lender's consumption-to-output ratio,  $\frac{\tilde{C}}{\tilde{Y}} = 1 - \delta \frac{K}{\tilde{Y}} = 1 - \frac{\delta \alpha \tilde{\beta}}{1 - \tilde{\beta}(1-\delta)}$ . Equation (12) implies  $\gamma \frac{Y}{L} (1-\rho) \tilde{C}^{-\sigma_L} = b (\bar{L} - L)^{-\sigma_W}$ , which uniquely solves for the steady-state allocation of land between the two agents because the left-hand side (LHS) is decreasing in the borrower's land holding  $L$ ,  $\lim_{L \rightarrow 0} LHS = \infty$ , and the right-hand side (RHS) is increasing in it,  $\lim_{L \rightarrow \bar{L}} RHS = \infty$ .

In the "first-best" allocation, the dynamics of the model is very similar to that of a standard RBC model with CUWJ preferences. There is no hump-shaped cyclical propagation mechanism in such a model for realistic parameter values on the lender side (e.g., the parameter values in Table 1). To see this intuitively, notice that the above program is a standard RBC model with

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<sup>15</sup>Since the lender is more patient with a lower discounting rate, we must have  $\tilde{C} > C$  in the steady state.

two consumption goods,  $\{C, \tilde{C}\}$ , except the relative price of  $C$  is infinity in the steady state. Hence, near the steady state we can ignore the weight of the borrower's consumption in the utility function by setting  $C_t = 0$ . The lender's land  $\tilde{L}$  in utility plays the role of leisure and the borrower's land  $L$  in the production function plays the role of hours worked. The aggregate land supply  $\bar{L}$  is equivalent to time endowment. Therefore, as in a standard RBC model with consumption habit, a one-time shock to productivity will not generate persistence in aggregate output although investment is more volatile than the case without habit.

### 2.3 Competitive Equilibrium with Borrowing Constraints

Denoting  $\tilde{\Lambda}_t$  as the Lagrangian multiplier of the constraint (1), the first-order conditions of the lender with respect to consumption, land investment, and lending are given, respectively, by

$$\tilde{C}_t^{-\sigma_L} = \tilde{\Lambda}_t \quad (13)$$

$$Q_t \tilde{\Lambda}_t = \tilde{\beta} E_t Q_{t+1} \tilde{\Lambda}_{t+1} + \tilde{\beta} b \tilde{L}_{t+1}^{-\sigma_W} \quad (14)$$

$$\tilde{\Lambda}_t = \tilde{\beta} E_t (1 + R_{t+1}) \tilde{\Lambda}_{t+1}. \quad (15)$$

Denoting  $\{\Lambda_t, \Phi_t\}$  as the Lagrangian multipliers of constraints (5) and (7), respectively, the first-order conditions of the borrower with respect to consumption, land investment, capital investment, and borrowing are given, respectively, by

$$[C_t - \rho C_{t-1}]^{-\sigma_B} = \Lambda_t \quad (16)$$

$$Q_t \Lambda_t = \beta E_t Q_{t+1} \Lambda_{t+1} + \beta \gamma E_t \frac{Y_{t+1}}{L_{t+1}} \Lambda_{t+1} + \theta_t \Phi_t E_t Q_{t+1} \quad (17)$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \quad (18)$$

$$\Lambda_t = \beta E_t (1 + R_{t+1}) \Lambda_{t+1} + \Phi_t E_t (1 + R_{t+1}). \quad (19)$$

A competitive equilibrium is a sequence of allocations  $\{C_t, \tilde{C}_t, B_{t+1}, K_{t+1}, L_{t+1}, \tilde{L}_{t+1}\}_{t=0}^{\infty}$  and prices  $\{Q_t, R_t\}_{t=0}^{\infty}$  such that: (i)  $\{C_t, \tilde{C}_t, B_{t+1}, K_{t+1}, L_{t+1}, \tilde{L}_{t+1}\}_{t=0}^{\infty}$  satisfies the first-order conditions (13)-(19), the transversality conditions,  $\lim_{t \rightarrow \infty} \beta^t \Lambda_t L_{t+1} = 0$ ,  $\lim_{t \rightarrow \infty} \beta^t \Lambda_t K_{t+1} = 0$ ,  $\lim_{t \rightarrow \infty} \tilde{\beta}^t \tilde{\Lambda}_t \tilde{L}_{t+1} = 0$ , and the complementarity condition,  $\Phi_t [\theta_t Q_{t+1} L_{t+1} - (1 + R_{t+1}) B_{t+1}] = 0$  for all  $t \geq 0$ , given  $\{Q_t, R_t\}_{t=0}^{\infty}$  and the initial endowments  $L_0 \geq 0, \tilde{L}_0 \geq 0, B_0 \geq 0, K_0 \geq 0$ ; (ii)

The good and asset markets clear for all  $t$ ,  $C_t + \tilde{C}_t + K_{t+1} - (1 - \delta)K_t = Y_t$  and  $L_t + \tilde{L}_t = \bar{L}$ , respectively.

The model has a saddle-path steady-state equilibrium in which the borrower is credit-constrained, i.e., equation (7) binds for all  $t$ . We abstract from any corner solutions with zero credit and, for simplicity, we assume that the steady state value of  $\theta_t = 1$  in the benchmark model.<sup>16</sup> In the steady state, equation (15) indicates that the interest rate is determined by the lender's time discounting factor,  $1 + R = \tilde{\beta}^{-1}$ . This interest rate of loanable funds is lower than the return determined by the firm's marginal product of capital. Equation (19) then implies  $\Phi = (\tilde{\beta} - \beta)\Lambda > 0$ , suggesting that the borrowing constraint binds around the steady state.<sup>17</sup> Equation (18) implies that the capital-to-output ratio is given by  $\frac{K}{Y} = \frac{\beta\alpha}{1-\beta(1-\delta)}$ . The capital-to-output ratio determines the return from capital, which is equal to the loanable funds rate if  $\beta = \tilde{\beta}$ ; or, as in the first-best economy, if there exists perfect risk sharing without borrowing constraints.<sup>18</sup> Since  $\theta = 1$ , equation (17) implies  $Q = (1 - \tilde{\beta})^{-1}\beta\gamma\frac{Y}{L} = \sum_{j=0}^{\infty} \tilde{\beta}^j \beta\gamma\frac{Y}{L}$ , suggesting that the price of land is determined by the present value of its marginal products. If  $\theta < 1$ , the price of land,  $Q = (1 - \beta - \theta(\tilde{\beta} - \beta))^{-1}\beta\gamma\frac{Y}{L} > (1 - \tilde{\beta})^{-1}\beta\gamma\frac{Y}{L}$ , is adjusted upward by the loan-to-collateral ratio because, other things equal, a tighter credit constraint increases the incentive for accumulating land so as to relax the constraint. The lender's budget constraint implies  $\tilde{C} = (1 - \tilde{\beta})QL = \beta\gamma Y$ , suggesting that the lender's consumption level is just the interest income, which is proportional to aggregate output. The borrower's budget constraint implies  $C + [\delta K + \beta\gamma Y] = Y$ , where the bracketed term denotes savings and part of the savings,  $\beta\gamma Y = (1 - \tilde{\beta})Q$ , is used to finance the loan and equals the lender's interest income (or the user's cost of financial capital). This indicates that the lender serves essentially as a bank and the borrower's total business investment can deviate from own savings because of bank's credit lending. In addition, since the value of  $\gamma$  is small, lender's consumption share ( $\beta\gamma$ ) will be small, so lender does not play a direct role in aggregate consumption and this is what we have in mind for the financial sector. All of the great ratios (e.g., capital-to-output ratio, land-to-output ratio, consumption-to-output ratio) are determined as functions of the model's structural parameters only. Once the steady-state distribution of land is determined, the steady-state values of all

<sup>16</sup>Our results remain qualitatively the same if  $\theta < 1$  (see section 3).

<sup>17</sup>In a model similar to ours, Iacoviello (2005) uses numerical methods to show that the probability of a non-binding borrowing constraint is very small even sufficiently away from the steady state.

<sup>18</sup>The gap between the capital rate of return and the loan rate in the steady state reflects a premium or wedge created by borrowing constraints.

other variables are determined through the great ratios. Because equation (17) is the demand curve of land and equation (14) gives the supply curve of land, the steady-state distribution of land across agents is determined uniquely by the implicit equation,

$$\beta\gamma\frac{Y(L)}{L} = \tilde{\beta}b(\bar{L} - L)^{-\sigma_W} \tilde{C}(L)^{\sigma_L}, \quad (20)$$

where the left-hand side decreases in  $L$  and the right-hand side increases in  $L$ .

## 2.4 Quantitative Implications

The model's stationary equilibrium path is solved by log-linearizing the model around the interior steady state (see the equations in the appendix). The existence of a unique rational expectations equilibrium can be confirmed by the eigenvalue method. As in KM and others in this literature,<sup>19</sup> we examine the dynamics of the model near the steady state after a sudden unexpected shock to financial conditions ( $\theta$ ) or TFP ( $A$ ), assuming that the borrowing constraint always binds near the steady state.

Table 1. Parameter Values

Parameter	$\theta$	$\tilde{\beta}$	$\delta$	$\alpha$	$\gamma$	$\rho$	$\sigma_L$	$\sigma_W$	$\sigma_B$	$\beta$	$\rho_A$
Calibration 1	1	0.99	0.025	0.35	0.05	0.9	0(1)	0(1)	4	0.5	0
Calibration 2							0(1)	0(1)	2	0.8	0.9

*Calibration.* The time period is a quarter. As a benchmark, we set the lender's discounting factor  $\tilde{\beta} = 0.99$  (implying a 4% annual interest rate), the rate of capital depreciation  $\delta = 0.025$ , capital's income share  $\alpha = 0.35$ , land share  $\gamma = 0.05$ , and the utility weight parameter  $b$  is set so that the steady-state ratio of land allocated between the two types of agents  $\frac{\bar{L}}{L} = 1$ . The results are not very sensitive to these particular parameter values (i.e., 1-10% changes in these values give similar results).<sup>20</sup> The risk aversion parameters for the lender,  $\{\sigma_L, \sigma_W\}$ , determine the volatility of both the interest rate and the asset price and they are hence left free for experiments. The shape of the impulse responses are sensitive to several key parameters, including the degree of habit persistence  $\rho$ , the borrower's discounting factor  $\beta$ , and risk aversion  $\sigma_B$ . Ravina's (2007, Table 6 and Table 8) empirical estimates based on household data show that the combined coefficient of both internal and external habit formation is around  $0.8 \sim 0.94$ . In some case, the combined coefficient can exceed 0.95 (Ravina, 2007; Tables 10-13). The

<sup>19</sup>See, e.g., Kocherlakota (2000), Cordoba and Ripoll (2004a,b), and Iacoviello (2005).

<sup>20</sup>Under these parameter values, the implied steady-state consumption level of the lender is small, less than 5% of aggregate output.

parameter value for  $\rho$  around 0.9 is also consistent with the recent estimates of habit formation based on aggregate data (see, e.g., Chen and Ludvigson, 2009). Based on this literature, we set  $\rho = 0.9$  in our model as the benchmark value for consumption habit.

In general, the stronger the borrower's incentive to borrow, the more likely the credit cycle. The results are qualitatively similar under either TFP shocks or financial shocks. We present the effects of TFP shocks first and defer the discussion on financial shocks until Section 3 where we study credit cycles in several variants of the benchmark model. We experiment with two sets of values for the other two key parameters,  $\{\sigma_B, \beta\}$ . In the first set (calibration 1), we choose  $\sigma_B = 4$  and  $\beta = 0.5$ , and we assume the TFP shocks are *i.i.d.* In the second set (calibration 2), we choose  $\sigma_B = 2$  and  $\beta = 0.8$ , and in this case the TFP shock is persistent with an  $AR(1)$  coefficient  $\rho_A = 0.9$ . The calibrated parameter values are reported in table 1. Notice that in the first parameter set (calibration 1), the value of  $\beta$  is quite low. A low value of the time-discounting factor implies that the borrower has a strong incentive to increase saving in response to a higher social living standard. Consequently, only *i.i.d.* shocks are needed to trigger hump-shaped output dynamics. On the other hand, if  $\beta$  is relatively large, persistent TFP shocks are needed to generate hump-shaped output dynamics.<sup>21</sup> The parameter values are summarized in table 1.

*Impulse Responses.* The impulse responses of the model to an *i.i.d.* TFP shock (calibration 1 in table 1) are graphed in the top-row panel in figure 1 and those under a persistent  $AR(1)$  TFP shock (calibration 2 in table 1) in the bottom-row panel. The left-column windows in figure 1 show the responses of aggregate output ( $Y$ ), aggregate consumption ( $C + \tilde{C}$ ), aggregate capital formation ( $K_{t+1}$ ), and the borrower's land investment ( $L_{t+1}$ ) when the lender is risk neutral ( $\sigma_L = \sigma_W = 0$ ); and the right-column windows in figure 1 show the responses of aggregate output, aggregate consumption, the price of land ( $Q_t$ ), and the gross interest rate ( $R_t$ ) when the lender is risk averse:  $\sigma_L = \sigma_W = 1$ .

In the top-row panel, since the shock lasts for just one period with zero persistence, any serial correlation in the impulse responses is generated endogenously within the model. In the bottom-row panel, the hump-shaped dynamics also reflect endogenous propagation mechanisms because the TFP shock has only  $AR(1)$  monotonic persistence. Regardless of the persistence of the shock, with a risk neutral lender (left-column windows), the land price and interest rate in the model are constant; hence, credit-resource reallocations or debt fluctuations are driven entirely

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<sup>21</sup>With a low value of  $\beta$ , the agent discounts the future heavily and his/her future consumption is less than current consumption compared to the social living standard. This implies that raising future consumption can generate higher marginal utility than raising current consumption if the social living standard  $\tilde{C}$  increases. Hence, a lower value of  $\beta$  provides additional incentives to save under CUWJ preferences after a transitory shock.

by changes in the quantities of collateralized assets. Whereas with a risk averse lender (right-column windows), the land distribution across the lender and the borrower becomes constant but the land price and interest rate fluctuate; hence, credit-resource or debt reallocations are driven by the price of collateralized assets.<sup>22</sup>

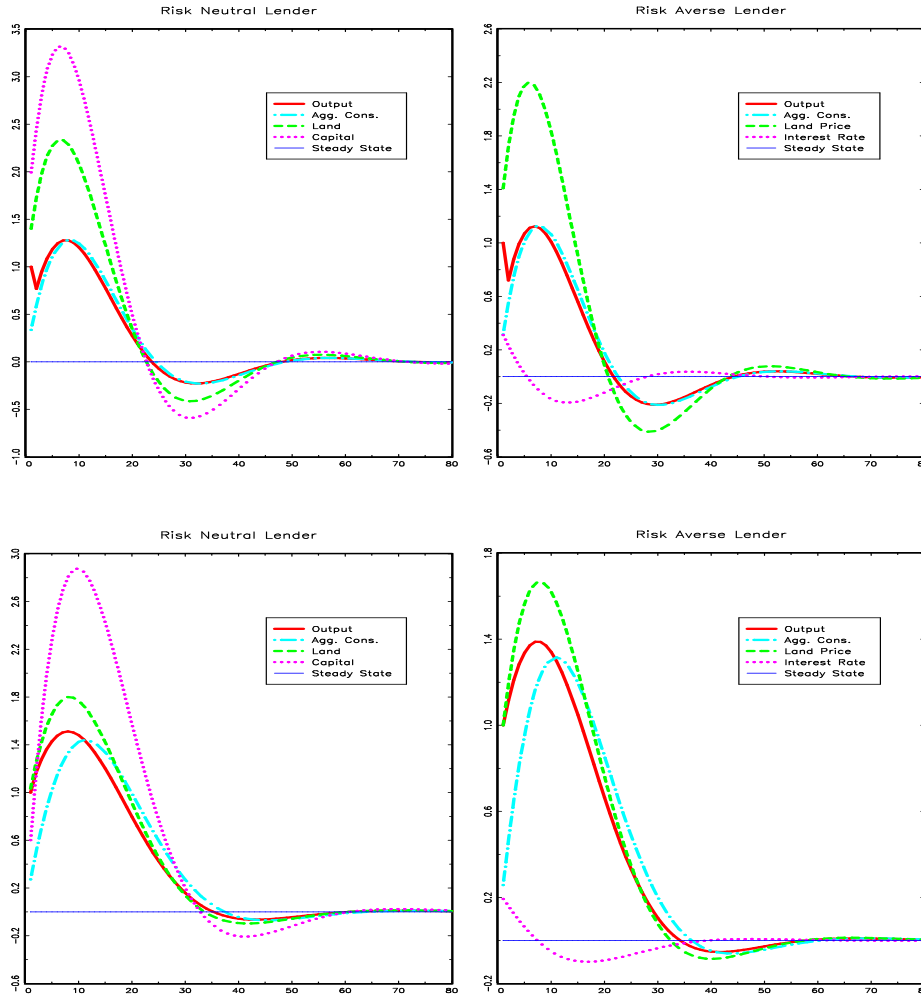


Figure 1. Impulse Response to TFP Shock.

More specifically, the top-left window in figure 1 shows that a purely transitory shock can generate highly persistent and hump-shaped fluctuations in aggregate activities, due to the presence of stable complex eigenvalues in the linearized system. The dynamic *multiplier-accelerator* effect on aggregate output reaches its maximum after 6 quarters of the shock and the increase in output at the peak is about 125% of the shock itself on TFP.<sup>23</sup> The economy

<sup>22</sup>A knife-edge case arises when lender's utility is logarithmic, which implies that the substitution effect and the income effect of an interest rate increase (that is triggered by a positive TFP shock) on lender's land savings cancel out, as explained in more details in Section 2.5.

<sup>23</sup>To see the dramatic difference between our model and that of Kiyotaki and Moore (1997), the readers may compare our figure 1 with their figure 3 (p.238).



over-shoots its steady state from above as it retreats from the initial boom and enters a recession before settling down on a long-run steady state via dampened cycles. New capital formation and land investment are excessively volatile and procyclical, suggesting that credit resources are rapidly pumped into the production sector from the financial system. The length of each boom-bust cycle is about 7 – 10 years long under the current parameterization.<sup>24</sup> Because the lender is risk neutral, the interest rate and land price do not change over time, albeit the marginal product of capital changes dramatically.<sup>25</sup> Thus, prolonged booms are possible without triggering high real interest rates and international financial-capital inflow.

The nature of the credit cycle is not sensitive to the degree of risk aversion of the lender. The top-right window in figure 1 shows that investment, output, and consumption fluctuate in the same manner with a similar magnitude and cyclical length when the lender’s risk aversion parameters are set to  $\sigma_L = \sigma_W = 1$ . In this case, the quantity of the collateralized asset (land) becomes constant but the land price starts to fluctuate violently, producing cyclical fluctuations in the credit limit. In addition, the real interest rate shows persistent decline during the boom period despite rising credit demand, consistent with the empirical observation of Caballero, Farhi, and Hammour (2006). Notice that land price is two times more volatile than output despite the interest rate being endogenous, in contrast with KM (1997) and CR (2004b) who assume a constant interest rate.

The multiplier-accelerator mechanism is preserved under the second parameter set (see the lower panel in figure 1), except that the initial hump is now much larger and the aftermath recession is less severe. A key feature of the model is that aggregate consumption is nearly as volatile as output while capital investment is excessively more volatile than output. For example, the standard deviation of consumption (investment) relative to output is 0.99 (20.6) under the first set of parameter values and 0.96 (8.3) under the second parameter set. These predictions are qualitatively consistent with the stylized facts documented by Mendoza and Terrones (2008). It is also possible to generate a less volatile investment if we re-calibrate the capital depreciation rate. For example, if we use the second set of parameter values but reset the depreciation rate to  $\delta = 0.05$ , then the implied relative volatility of consumption (investment) becomes 0.93 (4.8) while the cyclical pattern in figure 1 (lower-left window) is preserved.

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<sup>24</sup>Changing the parameter values can also change the length of the cycles in our model.

<sup>25</sup>The response of aggregate output on impact is one percent because all production factors are predetermined and there is no labor. In the second period and beyond, changes in output are completely driven by land and capital accumulations. There is a downward kink in output in the second period because the accumulated asset stocks are not large enough to completely offset the withdraw of the TFP shock.

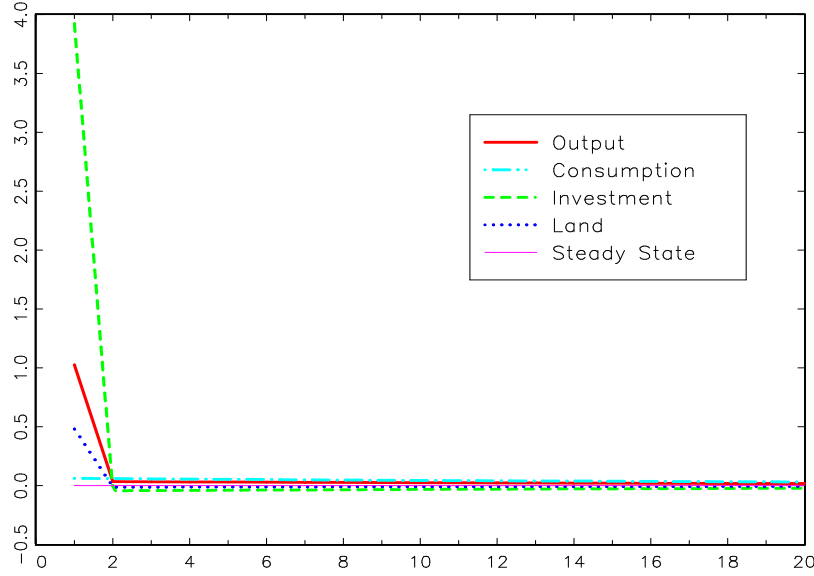


Figure 2. Impulse Responses in a First-Best Allocation.

As a comparison, the impulse responses of the "first-best" allocation to a one-time positive shock to TFP are graphed in figure 2, where the parameter values are exactly the same as in the competitive equilibrium (top-right window in figure 1) with risk averse lenders (i.e.,  $\sigma_L = \sigma_W = 1$ ). It shows that the impact of the shock on output is not amplified, and it is short-lived with zero persistence. Although investment is more volatile than output, the capital stock is as smooth as consumption.<sup>26</sup>

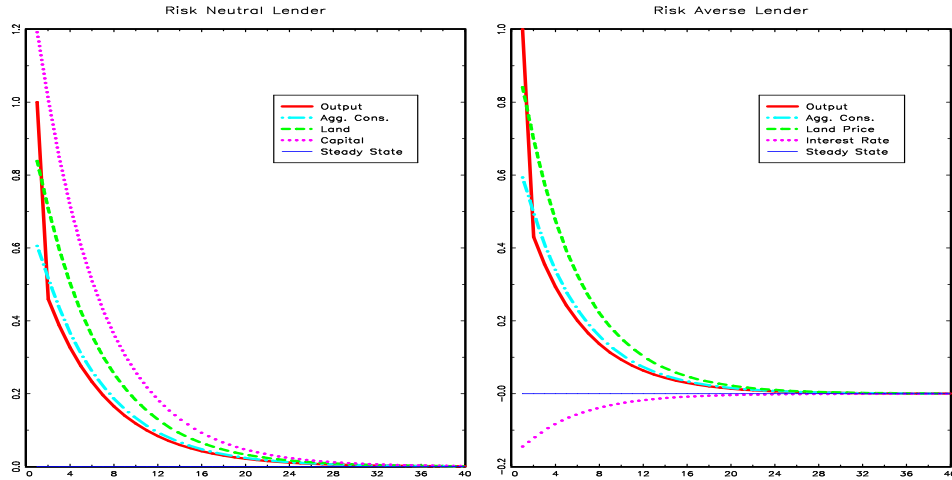


Figure 3. Impulse Responses to an i.i.d. TFP Shock without Habit ( $\rho = 0$ ).

<sup>26</sup> As changes of the capital stock, investment is a flow variable and is hence more volatile than capital in percentage terms. The log-linear relationship between investment and capital is given by  $i_t = \frac{1}{\delta} (k_{t+1} - (1 - \delta)k_t)$ . In the competitive equilibrium of our model, the capital stock is far more volatile than output, suggesting an even greater volatility of investment. Because movements in other variables appear to be trivial relative to investment, we plot the capital stock instead of investment series in figure 1.

Without consumption habit, the model has no hump-shaped credit cycles. For example, setting  $\rho = 0$  in the benchmark model leads to monotonic impulse responses to i.i.d. TFP shocks as shown in figure 3.

## 2.5 Dissecting the Mechanism

To understand the intuition behind the above results, especially the role played by CUWJ and collateral constraints, consider a simpler version of the basic model where the lender is risk neutral ( $\sigma_L = \sigma_W = 0$ ) and there is no capital. Risk neutrality implies a constant interest rate  $(1 + R) = \tilde{\beta}^{-1}$  and a constant land price  $Q$  according to equations (13)-(15). Equation (19) then becomes  $\Phi_t = \tilde{\beta}\Lambda_t - \beta\Lambda_{t+1}$ . Assume  $\sigma_B = 1$  and the borrowing constraint binds:

$$(1 + R)B_{t+1} = QL_{t+1}. \quad (21)$$

The leverage effect of collateralized borrowing modifies the borrower's budget constraint in the following way:

$$C_t + QL_{t+1} - (QL_t - (1 + R)B_t) = \tilde{\beta}QL_{t+1} + AL_t^\gamma, \quad (22)$$

where the third term on the left-hand side vanishes because the borrower sells the current land stock to repay the last-period debt, that is,  $QL_t = (1 + R)B_t$ . Therefore, the budget constraint can be rewritten as  $C_t + QL_{t+1} = \tilde{\beta}QL_{t+1} + Y_t$ , where the right-hand side is the sum of collateralized borrowing and output, while the left-hand side sums up consumption and land expenditure. Finally, the budget constraint simplifies to

$$C_t + Q(1 - \tilde{\beta})L_{t+1} = AL_t^\gamma, \quad (23)$$

where  $Q(1 - \tilde{\beta})L_{t+1}$  is the downpayment required to invest in land: whenever investing  $QL_{t+1}$ , the borrower is lent  $\tilde{\beta}QL_{t+1}$ . In other words, leveraged lending permits the borrower to finance investment at a level far exceeding his/her own savings because the downpayment is close to zero under our parameterization that  $\tilde{\beta}$  is close to one. Suppose there is no habit formation ( $\rho = 0$ ), this leveraged lending would imply that the borrower has a strong incentive to raise consumption when income increases, knowing that it is possible to finance investment largely through borrowing. This kills the boom-bust cyclical mechanism by discouraging investment.

To see this analytically, combine the first-order conditions (16)-(17) and (19) by eliminating  $\Phi_t$ , we get

$$Q(1 - \tilde{\beta})\frac{1}{C_t - \rho C_{t-1}} = \beta\gamma E_t \left[ \frac{Y_{t+1}}{L_{t+1}} \frac{1}{C_{t+1} - \rho C_t} \right]. \quad (24)$$

This equation determines the value of land in the steady state as the present value of the marginal products:  $Q = \frac{1}{1-\beta}\beta\gamma\frac{Y}{L}$ . Equations (21), (23), and (24) plus a standard transversality condition fully determine the dynamic equilibrium paths of  $\{C_t, L_t, B_t\}$  in this simple model.<sup>27</sup> When  $\rho = 0$ , the model has closed-form solutions, with the decision rules of consumption, debt, and land investment given by the simple relationships,

$$C_t = (1 - \beta\gamma) AL_t^\gamma, \quad (25)$$

$$B_{t+1} = \frac{\tilde{\beta}\beta\gamma}{(1 - \tilde{\beta})} AL_t^\gamma, \quad (26)$$

$$L_{t+1} = \frac{\beta\gamma}{(1 - \tilde{\beta})Q} AL_t^\gamma. \quad (27)$$

Notice that all decision variables are proportional to aggregate output. Log-linearizing the decision rules around the steady state gives  $c_t = b_{t+1} = l_{t+1} = \gamma l_t$ , where lower-case variables denote percentage deviations from the steady state. In this case, a one-percent increase in current output leads to a one-percent increase in the levels of both consumption and new debt, which in turn translates into a one-percent increase in land stock ( $L_{t+1}$ ) and a  $\gamma$ -percent increase in the next period's output. Thus, with the borrower as the single producer in the economy, a one-time shock to TFP can generate serially correlated movements in aggregate output with the degree of persistence determined by  $\gamma$ . This roughly explains the result obtained by Kocherlakota (2000) and Cordoba and Ripoll (2004a).

However, the  $\gamma$ -persistence is monotonic and there do not exist hump-shaped boom-bust cycles. That is, endogenous credit constraints, by themselves, generate endogenous persistence but do not give rise to the hump-shaped multiplier-accelerator mechanism.

When  $\sigma_L = \sigma_W = 0$ , both the land price  $Q$  and the interest rate  $R$  are constant by virtue of risk neutrality. This implies that when the borrower experiences a positive TFP shock, he is willing to borrow more and the only way to reallocate resources away from the lender is to increase the borrower's land holding (see equation (21)). If in contrast  $\sigma_L$  and  $\sigma_W$  are nonzero, both the land price and the interest rate increase at impact, because the borrower's demands for land and credit go up, which in itself relaxes his credit constraint. As a consequence, it is not immediate whether or not land reallocation is needed for generating boom-bust cycles. In fact, a knife-edge case arises when the lender has logarithmic utility (that is, when  $\sigma_L = \sigma_W = 1$ ), which implies that there is no land reallocation from the lender to the borrower after the shock

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<sup>27</sup>The lender's consumption level is simply determined by interest income.

hits the economy. However, with larger lender's risk aversion, the land stock, the land price and the interest rate will be cyclical, which would be an intermediate case between the left and right panels in Figure 1. This is because the lender is a net saver so that an interest rate increase has an ambiguous effect on his land savings. Not surprisingly, these effects cancel out only under logarithmic utility, which implies that there is land reallocation in response to an exogenous shock for an open set of parameter values.

In order to generate a more persistent and hump-shaped propagation mechanism, we need a larger fraction of the income to be saved and invested in each period, rather than being consumed. This is why the picture changes dramatically when there is consumption habit ( $\rho > 0$ ). Habit formation creates a strong incentive for the borrowers to save the transitory income so as to increase future consumption in the long run. With habit formation, agents are more interested in consumption growth than in the consumption level. Hence, after a TFP shock to income, the borrowers increase their marginal propensity to save, which provides more loanable funds for investment. This motive for wealth accumulation is reinforced by the borrowers' desires to borrow under the collateralized lending, thus they opt to invest as much as possible not only to ensure future consumption growth but also to raise the collateral value so as to further reduce the borrowing constraint. To see this, note that equation (24) indicates that with  $\rho > 0$  and holding tomorrow constant, a one-percent increase in consumption today due to a one-percent increase in income is no longer optimal because it decreases the left-hand side of (24) by more than one percent (due to the habit stock  $\rho C_{t-1}$ ) while the right-hand side would decrease by less than one percent after land investment ( $L_{t+1}$ ) raises by one percent (due to the rise in the habit stock  $\rho C_t$ ). Hence, to reach an equilibrium, consumption should increase by less than one percent and land investment should increase by more than one percent. This higher investment level will bring about not only more output next period but also more credit by relaxing the borrowing constraint in the current period. Thus, the incentive for saving under habit formation and the motives for investment under leveraged lending start to reinforce each other dynamically, making possible a cumulative process of output expansion and investment boom that underlies a persistent and hump-shaped propagation mechanism.

However, because of diminishing marginal product of capital, over-investment is not sustainable by aggregate savings and a rising debt level will ultimately erode the borrowers' aggregate demand (consumption and investment), resulting in a collapse of the "bubble" followed by a recession. In the downturn phase, the sluggish behavior of consumption under habit implies insufficient savings than needed to prevent the economy from a "soft landing", forcing the economy to overshoot and converge back to steady state in a cyclical fashion. Therefore, output falls below its long run level for a while so that a recession inevitably follows the investment

boom.

### 3 Robustness Analysis

This section shows that our main results are robust to financial shocks as well as to extensions of the benchmark model, including (i) symmetric agents in all dimensions except the discounting factor, (ii) small open economy, and (iii) elastic labor supply. We call the model with symmetric agents Model 2, the model with a small open economy Model 3, and the model with endogenous labor Model 4.

In Model 2, the lender and the borrower are now symmetric in all dimensions except the discounting factor. In particular, the lender's problem becomes

$$\max E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ \frac{[\tilde{C}_t - \rho \tilde{C}_{t-1}]^{1-\sigma_L}}{1-\sigma_L} \right\}, \quad \sigma_L \geq 0; \quad (28)$$

$$\tilde{C}_t + \tilde{K}_{t+1} - (1-\delta)\tilde{K}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + B_{t+1} \leq (1+R_t)B_t + A_t \tilde{K}_t^\alpha \tilde{L}_t^\gamma. \quad (29)$$

where the budget constraint (29) replaces equation (1).<sup>28</sup>

In Model 3, a representative agent in the home country borrows from the rest of the world at the given (exogenous) interest rate  $R_t$  and solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[C_t - \rho \bar{C}_{t-1}]^{1-\sigma_B}}{1-\sigma_B} \right\}, \quad \sigma_B \geq 0; \quad (30)$$

$$C_t + K_{t+1} - (1-\delta)K_t + Q_t(L_{t+1} - L_t) + (1+R_t)B_t \leq B_{t+1} + A_t K_t^\alpha L_t^\gamma \quad (31)$$

$$(1+R_{t+1})B_{t+1} \leq \theta_t Q_{t+1} L_{t+1},$$

where the total supply of land is fixed at  $L_t = \bar{L}$ . There is also a new type of financial shocks hitting the economy, the world interest rate  $R_t$ .

In Model 4, we introduce endogenous labor supply into the basic model of Section 2. Because habit formation induces a strong negative income effect with standard separable preferences, labor supply decreases after a positive TFP shock. This is inconsistent with the data. In contrast, the absence of income effects ensures that labor is procyclical, in accord with the US

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<sup>28</sup>For simplicity, we have dropped land from the lender's utility.

data. For this reason, we follow Greenwood, Hercowitz, and Huffman (1988) by adopting the following utility function with no income effect:

$$\frac{1}{1 - \sigma_B} \left[ C_t - \rho \bar{C}_{t-1} - P \frac{N_t^{1+\eta}}{1+\eta} \right]^{1-\sigma_B}, \quad \eta \geq 0; \quad (32)$$

where  $N_t$  denote total hours worked of the representative borrower. The idea that the GHH utility function is needed to generate positive labor comovement under consumption habit has also recently been followed by Gertler, Kiyotaki, and Queralto (2010) and others. The aggregate production function becomes

$$Y_t = AK_t^\alpha L_t^\gamma N_t^{1-\alpha-\gamma}. \quad (33)$$

The elasticity of labor supply is  $\frac{1}{\eta}$ . It can be shown that the steady-state utility level is strictly positive only if the inequality,  $(1 + \eta)(1 - \rho) > (1 - \alpha - \gamma)\frac{Y}{C}$ , holds; which imposes constraints on the values of  $\rho$  and  $\eta$ . For example, if  $\rho$  is close to one, then  $\eta$  must be large to generate positive utility. This model reduces back to the benchmark model with fixed labor if  $\eta = \infty$ . This elasticity parameter of labor supply is set at  $\eta = 6$ , implying a labor supply elasticity of 0.17, which is consistent with the microeconomic literature's finding of a relatively small labor supply elasticity.

Table 2. Common Parameter Values

Parameter	$\eta$	$\theta$	$\tilde{\beta}$	$\delta$	$\alpha$	$\gamma$	$\sigma_W$	$\sigma_L$	$\sigma_B$	$\rho$	$\beta$	$\rho_A$	$\rho_\theta$	$\rho_R$
Benchmark	$\infty$	0.9	0.99	0.025	0.35	0.05	1	4	4	0.9	0.5	0	0.9	NA
Model 2							NA							NA
Model 3	NA						NA	NA						0.9
Model 4	6													NA

Model 2 (symmetric agents), Model 3 (small open economy), Model 4 (endogenous labor). Blank entry means same parameter value as in benchmark model (same column, top row). "NA" means "not applicable."

We adopt a consistent calibration across all models for all common parameters. Because the steady-state value of  $\theta$  (the loan-to-collateral ratio) amplifies all shocks (especially financial shocks), we set  $\theta = 0.9$  in all models so that the loan-to-collateral ratio is less than 1 in the steady state. The parameter values are summarized in Table 2.

Figure 4 shows the impulse responses of aggregate output in the different models to an i.i.d. TFP shock and AR(1) financial shocks. The benchmark model is presented in the top-left window, the symmetric agent model (Model 2) in the top-right window, the small-open-economy model (Model 3) in the lower-left window, and the endogenous labor model (Model 4) in the

lower-right window. To make the impulse responses quantitatively comparable in the figure, we have chosen a positive 1% standard deviation shock to TFP ( $A_t$ ) and to the loan-to-collateral ratio ( $\theta_t$ ), and a negative 1% standard deviation shock to the interest rate ( $R_t$ ).

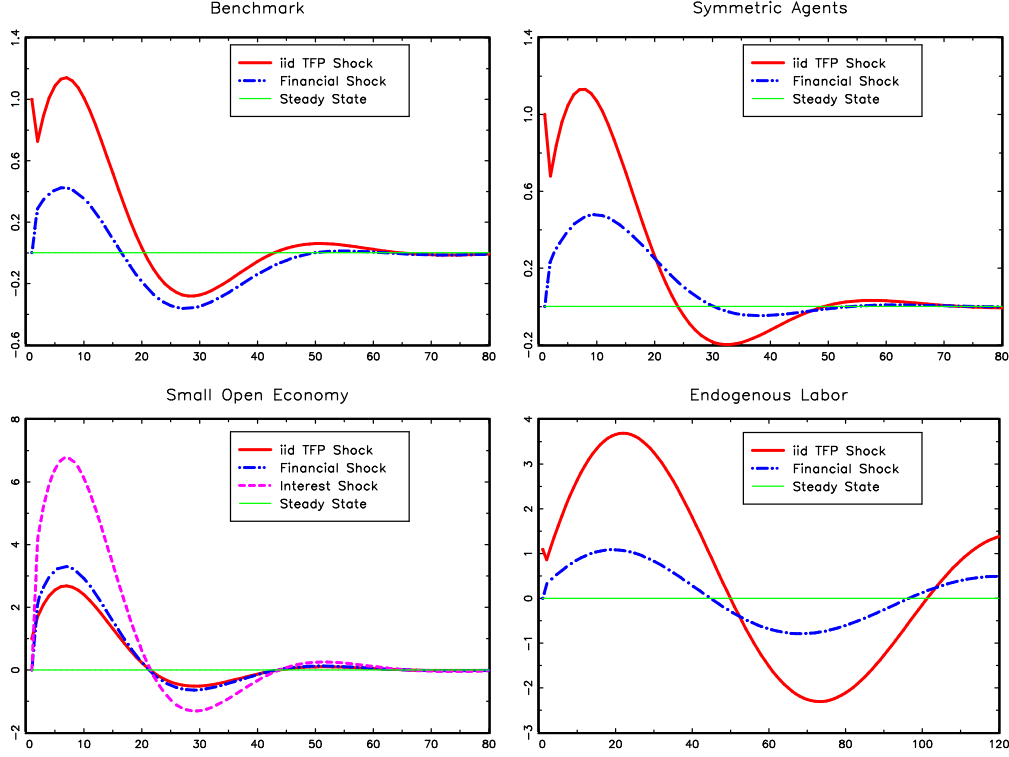


Figure 4. Responses of Aggregate Output to Shocks.

It is clear from the figure that shocks to both TFP and financial conditions can generate boom-bust cycles in both closed and open economies. The critical difference is that small-open economies are far more susceptible to boom-bust cycles than closed economies. In particular, aggregate output is 2 (or 10) times more volatile in the small-open economy than in the benchmark model under TFP (or financial) shocks. In addition, a negative world interest rate shock can also trigger a boom-bust cycle in a small open economy.

Another feature to notice in Figure 4 (lower-right window) is that endogenous labor amplifies the boom-bust cycle in terms of both magnitude and cycle length. For example, under either TFP or financial shocks, the magnitude of output at the peak of a boom and the length of the boom are nearly three times as large as those in the benchmark model with fixed labor.<sup>29</sup>

Capital investments in all models are many times more volatile than output under either TFP or financial shocks, consistent with the observed features of investment booms discussed

<sup>29</sup>Qualitatively similar results are also obtained under i.i.d. shocks to financial conditions. To conserve space, these results are not reported but available upon request.



by Mendoza and Terrones (2008). Because investment is so much more volatile than output, including it in the figure would obscure the hump-shaped output. But its volatility can be inferred from the response of capital stock (see, e.g., Figure 1).

The impulse responses of asset (land) price to TFP and financial shocks are presented in Figure 5. Among other things, two features are worth emphasizing: (i) the asset price is more volatile than output, (ii) it is more susceptible to financial shocks than to TFP shocks (except in the endogenous labor model), and (iii) the response of the asset price to shocks is hump-shaped and highly persistent in the benchmark model and in its variant with endogenous labor. The intuition is as follows.

First, financial shocks act essentially as an aggregate demand shock because they stimulate both consumption and asset demand without improving TFP. Hence, asset prices tend to increase more under financial shocks than under TFP shocks.

Second, equation (17) provides intuition on why the land price increases very little initially in the benchmark model compared with both the symmetric-agent model and the small-open-economy model. Consider financial shocks as an example. The last two terms on the RHS of equation (17) dominate the initial changes on the RHS and LHS of the equation after a shock. Because of consumption habit, changes in marginal utility of consumption  $\Lambda_t$  is small in both the initial period and the future periods, implying that  $Q_t$  absorbs most of the impact of shocks. However, since more land is reallocated to the lender in the benchmark model compared to Model 2 (with symmetric agents), the marginal productivity of land falls by more and the impact on  $Q_t$  is therefore dampened more in the benchmark economy (top-left window in figure 5) than in Model 2. In addition, a positive shock to the loan-to-collateral ratio relaxes the borrowing constraint and reduces the Lagrangian multiplier  $\Phi_t$ , which also partially cancels out the effect of the increase in  $\theta_t$  on the RHS of equation (17). However, the borrowing constraint does not relax as much in the symmetric-agent model (top-right window in figure 5) because the lender also needs land and capital to produce output. This implies that  $\Phi_t$  decreases less in Model 2 than in the benchmark model. In summary, the negative changes of land productivity and the Lagrangian multiplier dampen the positive effect of financial shocks on land price in both models but they are more pronounced in the benchmark model compared to Model 2. Hence, the land price  $Q_t$  on the LHS of equation (17) does not change very much on impact in the benchmark model. This subdued initial impact creates the hump-shaped impulse response in the benchmark model. Hump-shaped impulse response does not emerge in Model 2 because the initial impact of the shock on land price is large enough to dominate future responses.

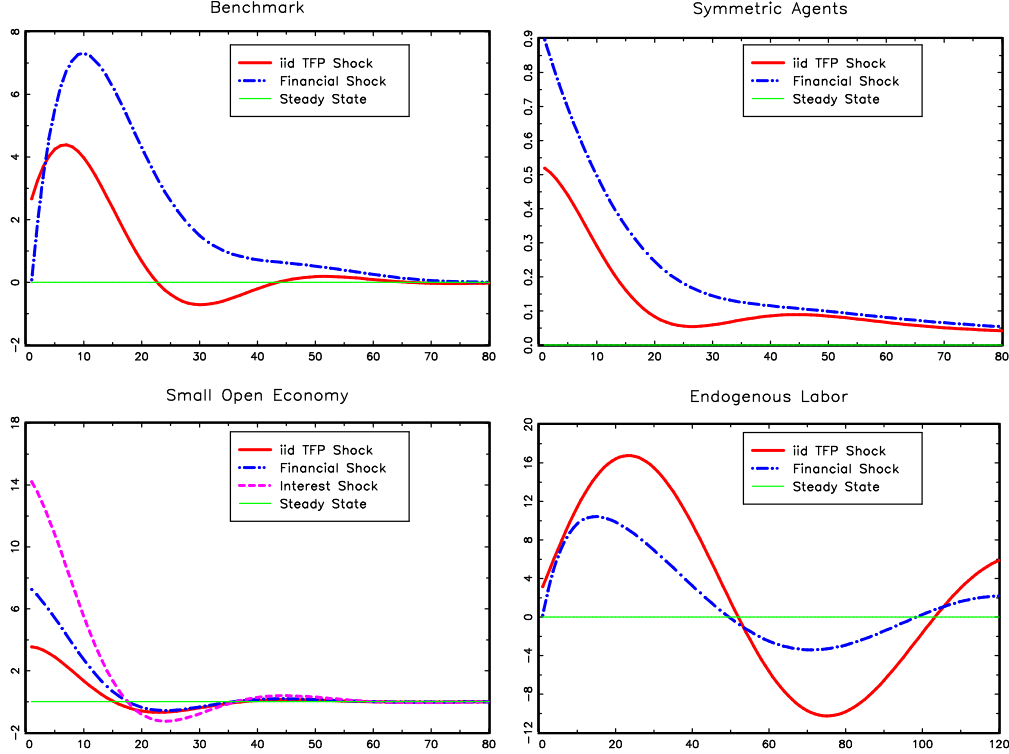


Figure 5. Responses of Asset Price to Shocks.

In the small-open-economy model (Model 3), land is in fixed supply, so the marginal product of land increases rather than decreases as in the other models. This explains the sharp rise in land price and its monotonic pattern in the bottom-right window in figure 5. Endogenous labor amplifies the impact of shocks and this is why the asset price is more volatile in Model 4 than in the benchmark model (but with similar cyclical patterns). To conserve space, we do not present impulse-responses of the interest rate, although they are available upon request. We simply note that the interest rate is more volatile under financial shocks than under TFP shocks in the models.

## 4 Policy Implications

The volatile nature of the boom-bust cycle calls for optimal stabilizing policies. However, if such policies exist, they must be time varying in nature (for more details, see our working paper, Pintus and Wen, 2008). Because time-varying policies are difficult to implement, practical policies in reality are often simple tax policies. To examine the effects of simple tax policies, figure 6 shows the impulse responses of aggregate output (in the economy with endogenous labor and risk neutral lender) to a one-time TFP shock (i.e., with no persistence) under different steady-state consumption tax rates. The other parameter values are the same as in table 1

(calibration 1), with  $\sigma_W = \sigma_L = 1$  and  $\eta = 6$  as in table 2. The results show that, as the tax rate increases, aggregate output is gradually stabilized with smaller amplification and lower persistence. Therefore, a constant-rate consumption tax does have stabilization effects when the tax rate is high enough. The intuition for the stabilization effect is that consumption tax discourages current and future consumption demand, which reduces the incentive for borrowing, hence mitigating the multiplier-accelerator effects of the credit constraints on investment. Similar results can also be obtained under income tax policies.

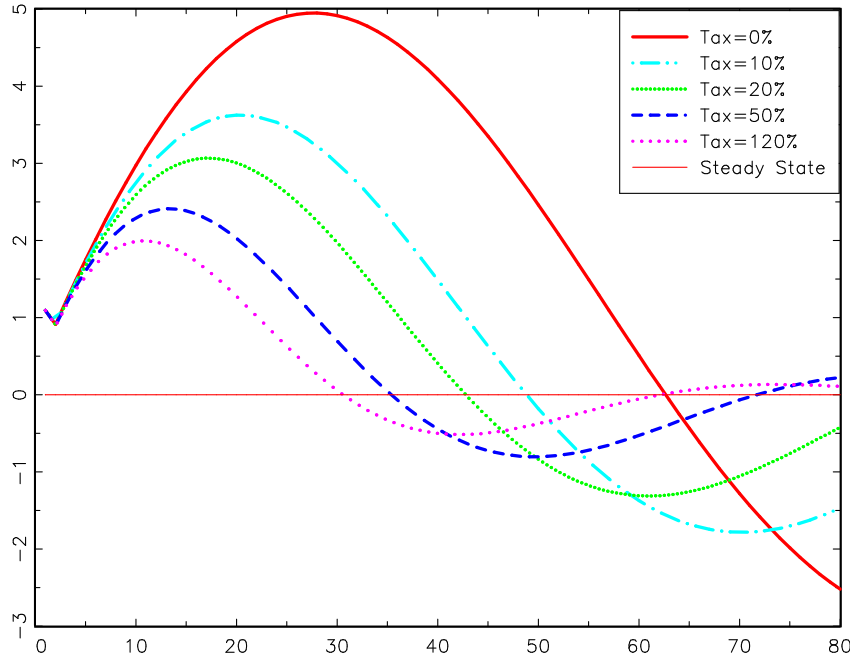


Figure 6. Stabilization Effects of a Consumption Tax.

However, simple tax policies cannot achieve the "first-best" allocation, more often they also introduce further distortions into the economy. As an example, we examine the business cycle effects of a sudden, unexpected, (one-period) 1% income-tax cut on the competitive economy with labor. Such a tax reduction is meant to boost the economy by increasing the after-tax marginal rates of return to work and investment. However, we show that such policies intended to stimulate the economy can be counter-productive and generate a long-period of recession instead of a boom.

Consider a standard income tax  $\tau_t$  on aggregate output  $Y_t$ . The borrower's resource constraint becomes

$$C_t + Q_t(L_{t+1} - L_t) + K_{t+1} - (1 - \delta)K_t + (1 + R_t)B_t \leq B_{t+1} + (1 - \tau_t)AK_t^\alpha L_t^\gamma N_t^{1-\alpha-\gamma} + T_t, \quad (34)$$

where  $T_t = \tau_t Y_t$  is a lump-sum transfer payment. Suppose the steady-state income tax rate is 20%; then a one-percent sudden decrease in the income tax rate has the following dynamic

effects shown in Figure 7.

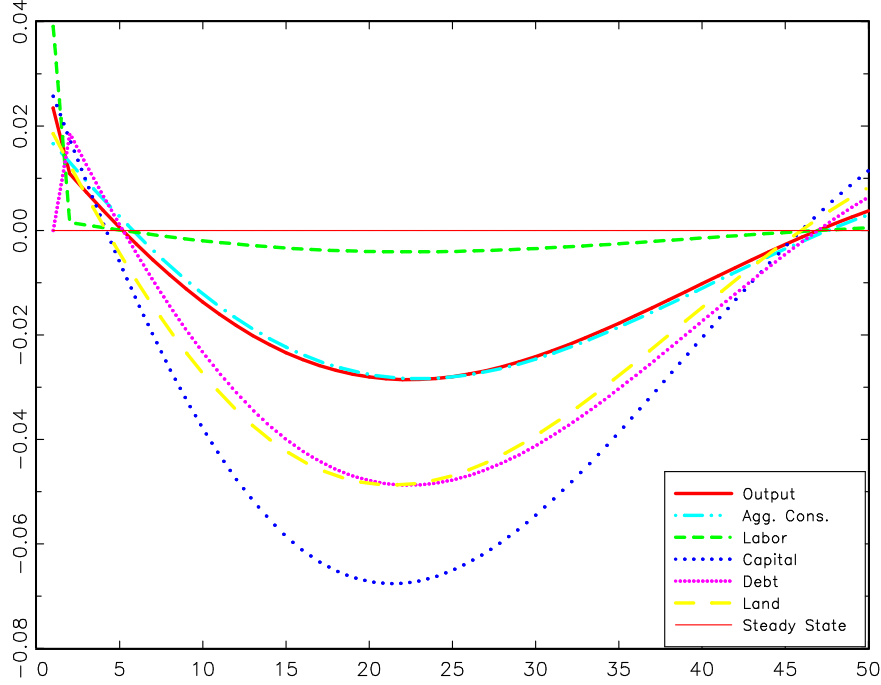


Figure 7. Impulse Responses to an Income-Tax Cut.

The intuition for the prolonged recession caused by a tax cut is as follows. Initially, a tax cut increases the incentives for working and investing. Hence, there is a short boom in the initial period in aggregate consumption, investment, labor, and output. However, since TFP has not changed, the increase in output is fully due to higher labor supply. Also, because the tax cut is financed by an equal decrease in the lump-sum transfer, the initial increase in aggregate demand is supported heavily by borrowing. Therefore, the debt level increases sharply in the second period and it chokes off investment because the marginal product of capital is below the loan rate. As investment decreases in the second period, the multiplier-accelerator mechanism kicks in and generates a cumulative process of contraction. Therefore, the stimulative package of a tax cut is counter-productive.

## 5 Conclusion

We argue that a simple neoclassical model with consumption habit and credit constraints accounts for the most salient features of typical credit-fueled investment booms documented by Mendoza and Terrones (2008). The model also offers an alternative way to rationalize the speculative-growth episodes stressed by Schneider and Tornell (2004), Caballero, Farhi and Hammour (2006). In general equilibrium, consumption growth crowds out savings and raises

the real interest rate, yet investment requires savings to finance with low capital costs. Hence, periodic boom-bust cycles featuring increases in consumption and investment (i.e., comovements) and their simultaneous collapses are difficult to generate in standard models without periodic movements in TFP or multiple steady states. Using a two-agent RBC model featuring a productive borrower who is credit-constrained but has strong incentives to accumulate wealth by saving and an unproductive lender who hoards "idle" resources but is willing to lend, this paper shows that dynamic interactions between the two forces create a cyclical mechanism that is broadly consistent with the cyclical behavior of the aggregate economy during a credit boom. In addition, our analysis indicates why neither of these two forces, in isolation, can generate such a cyclical pattern.

Our results reinforce the findings of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) that leveraged borrowing may have sizeable aggregate consequences. This may help figuring out not only why lowered credit standards in the subprime mortgage market designed to meet persistent housing demand from low-income households could have been responsible for the recent financial turmoil in the United States, but also why developing countries (where the supply of credit is severely constrained yet at the same time highly elastic because of endogenous credit limits based on collateral values, insider dealing, corruption, weak corporate governance, and speculative international capital flows) are more volatile and cyclical than developed countries.

Although we have shown that our results are robust to several extensions, including a small-open economy model, further work is called for to provide more microfoundations on the lender side as a genuine financial intermediary (perhaps along the lines of, e.g., Diaz-Jimenez et al., 1992). It also remains to be studied whether the larger volatility of our open-economy variant under financial shocks explains better the data than alternative models. Another line of future research would be to test the link between consumption habit and the business cycle. Micro studies (such as Ravina, 2007) have already shown that habits are important for household consumption behaviors. The question left is to show whether they are also empirically important for the business cycle. One insight provided by our model is that lagged consumption is an important state variable that affects how credit demand, aggregate output, investment, and asset prices move over the business cycle. In light of this, one possible way to address the aforementioned question is to extend Wen's (2007) analysis to detect through Granger causality test how aggregate financial and real variables interact both in the data and in the model. We believe this calls for future research.

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## 6 Appendix

The purpose of this appendix is to report the linearized version of the equations describing the competitive equilibrium with borrowing constraints (see section 2.3). In all equations below,  $x_t$  denotes the deviation of  $X_t$  from its steady-state value in percentage terms. For example,  $k_t \equiv (K_t - K)/K$ , where  $K$  is the steady-state capital stock. Eliminating  $R_{t+1}$  and  $\Phi_t$  by using respectively (15) and (19), the linearized versions of (1), (3), (4), (5), (7), (13)-(14), and (16)-(18) are respectively:

$$\frac{\tilde{C}}{B}\tilde{c}_t + \frac{Q\tilde{L}}{B}(\tilde{l}_{t+1} - \tilde{l}_t) + b_{t+1} = (1 + R)(b_t + \tilde{\lambda}_{t-1} - \tilde{\lambda}_t) \quad (35)$$

$$y_t = a_t + \alpha k_t + \gamma l_t \quad (36)$$

$$\tilde{l}_t = -\frac{L}{\tilde{L}}l_t \quad (37)$$



$$\frac{C}{Y}c_t + \frac{K}{Y}k_{t+1} - (1 - \delta)\frac{K}{Y}k_t + \frac{QL}{Y}(l_{t+1} - l_t) + \frac{(1 + R)B}{Y}b_t = \frac{B}{Y}b_{t+1} + a_t + \alpha k_t + \gamma l_t \quad (38)$$

$$\tilde{\lambda}_t - \tilde{\lambda}_{t+1} + b_{t+1} = \theta_t + q_{t+1} + l_{t+1} \quad (39)$$

$$-\sigma_L \tilde{c}_t = \tilde{\lambda}_t \quad (40)$$

$$q_t + \tilde{\lambda}_t = \tilde{\beta}(q_{t+1} + \tilde{\lambda}_{t+1}) - (1 - \tilde{\beta})\sigma_W \tilde{l}_{t+1} \quad (41)$$

$$\sigma_B(c_t - \rho c_{t-1}) = -(1 - \rho)\lambda_t \quad (42)$$

$$q_t + (1 - \theta\tilde{\beta})\lambda_t = \beta(1 - \theta)(q_{t+1} + \lambda_{t+1}) + \frac{\beta\gamma Y}{QL}(\lambda_{t+1} + y_{t+1} - l_{t+1}) + \theta\tilde{\beta}(q_{t+1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t) \quad (43)$$

$$\lambda_t = \beta(1 - \delta)\lambda_{t+1} + \frac{\alpha\beta Y}{K}(\lambda_{t+1} + y_{t+1} - k_{t+1}) \quad (44)$$